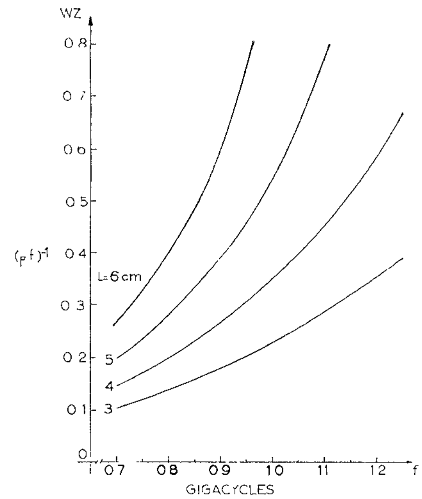

Fig. 3—Design curve ωZ vs f. Resonator line characteristic impedance is 50 Ω.

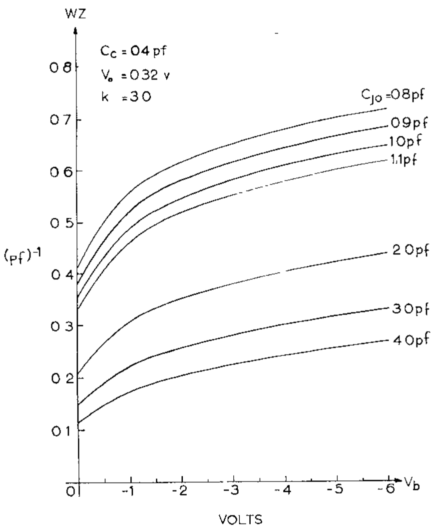

Fig. 4—Design curve ωZ vs V_b.

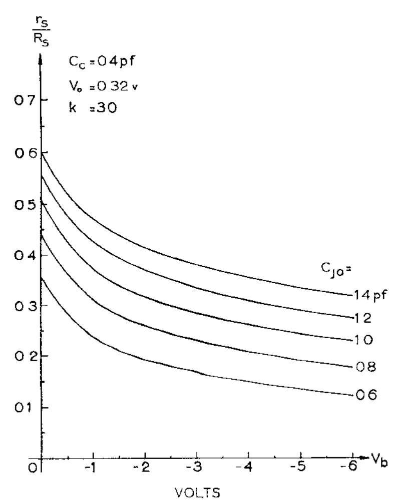

Fig. 5—Design curve r_s/R_s vs V_b.

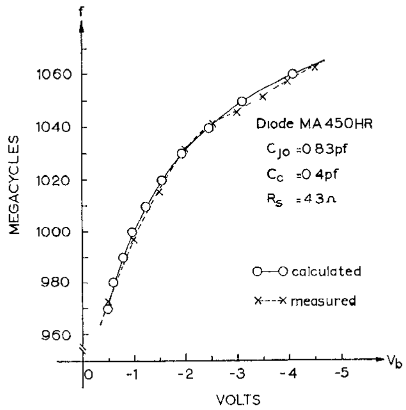

Fig. 6—Experimental results; tuning curve.

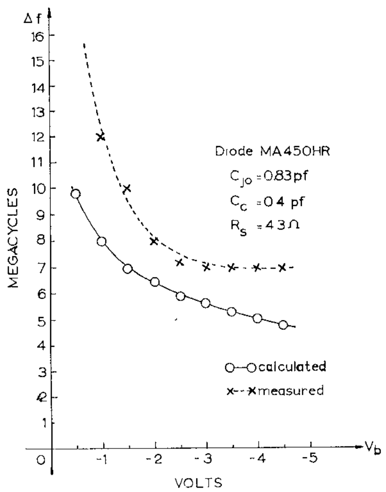

Fig. 7—Experimental results; bandwidth.

can be derived. The third curve is a plot of the quantity r_s/R_s as a function of the bias voltage V_b with the zero bias junction capacity C_{j0} as a parameter. The curve shown in Figs. 3 through 5 have been constructed for one series of diodes, the MICROWAVE ASSOCIATES MA 450 series which consists of eight diode models, all having the same C_c, V_0, and k.

To use the design curves, the procedure is as follows. Given a certain frequency range to cover, *e.g.*, f_1 to f_2, there are five diode and two resonator parameters to be selected. The diode parameters are C_{j0}, C_c, k, V_0 and V_m, the diode reverse bias breakdown voltage; the resonator parameters are R_0 and l.

A convenient design sequence is as follows:

1) Choose a particular series of diodes; this specifies C_c, V_0, and k.
2) Choose a value for the resonator half length l and characteristic impedance R_0.
3) Refer to the design curve ωZ vs V_b (Fig. 3) and read the two values of ωZ: $(\omega Z)_1$ and $(\omega Z)_2$ for the chosen l at the two frequencies f_1 and f_2.
4) Refer to the design curve ωZ vs V_b (Fig. 4) along the lines $(\omega Z)_1$ and $(\omega Z)_2$ and determine the C_{j0} values whose curves for a fixed value of C_{j0} interest both of the curves $\omega Z = (\omega Z)_1$ and $(\omega Z)_2$.
5) Compare the C_{j0} values with the values of available diodes. Failure to obtain an applicable value of C_{j0} will necessitate a second try by changing l or C_c or both.
6) Determine the resonator bandwidth Δf from the r_s/R_s vs V_b curve (Fig. 5) and the relation

$$\Delta f = \frac{f_0}{Q_u} = 2\pi f_0^2 r_s C_t. \tag{9}$$

Figs. 6 and 7 are typical results obtained with the above procedure. The desired frequency range was 970 Mc to 1060 Mc; also, the resonator half length l was 5 cm.

The predicted curves are for the unloaded resonator while the measured curves are for the complete filter (resonator and coupled loads). The generator and load impedance were each 50 Ω. The closeness of the measured and predicted tuning curves indicates that the coupling coefficients between the resonator and the 50 Ω coaxial loads were very small. The influence of coupling can be estimated by comparing the two bandwidth curves; the difference between the curves can be interpreted through (5).

It has been shown that the design of a UHF varactor-tuned coaxial filter can be conveniently accomplished through the use of design curves which characterize the diode. The design sequence is useful for determining the diode specifications necessary for a given tuning range and bandwidth.

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The Equivalent Circuit of a Plasma

Several recent papers, *e.g.*, Kaufmann and Steier,[1] show how the plasma resonance can be used to achieve rejection and transmission filters.

The object of this communication is to derive the equivalent circuit of two typical configurations and to discuss the bandwidth of the resultant filter. The equivalent dielectric constant of plasma at radial frequency ω is known to be

$$\epsilon_p = \epsilon_0 \left(1 - \frac{\omega_p^2}{\omega(\omega - j\nu)}\right)$$

where the symbols have the usual meaning. Consider a slab of uniform plasma of thickness d between two plates of contact area A;

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[1] I. Kaufman and W. H. Steier, "A plasma-column band-pass microwave filter," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 431–439; November, 1962.

then admittance across the plates neglecting fringing is

$$Y = j\omega \frac{A\epsilon_p}{d}$$
$$= \frac{A\epsilon_0}{d}\left\{\frac{\nu\omega_p^2}{\omega^2+\nu^2} + j\omega\left(1 - \frac{\omega_p^2}{\omega^2+\nu^2}\right)\right\}.$$

The equivalent circuit of this is clearly as shown in Fig. 1. C is the free space capacitance between the plates. When $\nu \ll \omega$, this circuit will give parallel resonance at $\omega = \omega_p$ and its $Q = \omega/\nu$.

Series resonance could be obtained by placing either a capacitance or an inductance in series with this circuit. In practice, the plates will have to be insulated from the plasma, thus adding a series capacitance C_1; the resultant circuit will show series resonance at $\omega < \omega_p$, resonant angular frequency being

$$\omega_r = \omega_p \sqrt{\frac{C}{C + C_1}}.$$

Since in practice C_1 is likely to be much greater than C, $\omega_r \to \omega_p\sqrt{C/C_1}$. This is the effect obtained by Yeung and Sayers.[2]

The effect of C_1 could be reduced by a small series capacitance and one could even make $\omega_r > \omega_p$ by adding a suitable series tuned circuit; however, this would seriously reduce the control of resonance by electron number density.

The bandwidth of such a circuit would depend on the relative value of generator and load resistance R_G and R_L, respectively. The ratio of center frequency to bandwidth is

$$Q = \frac{1}{\frac{\nu}{\omega_r} + \frac{(R_G + R_L)C\omega_p^2}{\omega_r}}$$
$$= \frac{1}{\frac{\nu}{\omega_r} + (R_G + R_L)C_1\omega_r}.$$

Clearly, in order to make this ratio large, C_1 and hence C must be small, *i.e.*, A should be small and d large.

Consider now the case of a plasma cylinder placed centrally across a waveguide and parallel to the broad side, the guide carrying a wave in TE_{10} mode. Bryant and Franklin[3] have shown that, neglecting the effect of glass wall, the admittance of such a plasma column placed in a matched guide is to a first approximation

$$Y_r = j2Y_0 \frac{\pi r^2 k_0^2}{bk_g} \cdot \frac{\epsilon_p - \epsilon_0}{\epsilon_p + \epsilon_0}.$$

where r is the plasma radius b is narrow guide dimension and Y_0 is the characteristic admittance of the guide.

Applying the above given expression for ϵ_p, we get

[2] T. H. Yeung and J. Sayers, *Proc. Phys. Soc. (London)*, vol. B70, pp. 663–668; 1957.

[3] G. H. Bryant and R. N. Franklin, *Proc. Phys. Soc. (London)*, vol. 81, pp. 531–543 and 790–792; 1963.

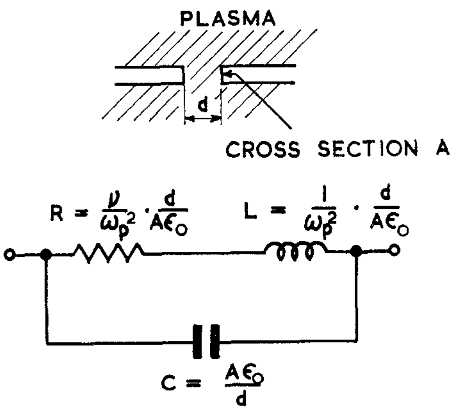

Fig. 1—The equivalent impedance of two electrodes immersed in plasma but insulated from it.

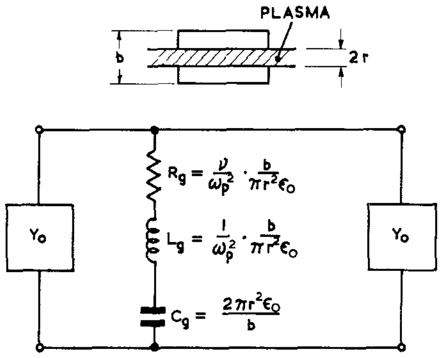

Fig. 2—The equivalent impedance of a plasma-filled tube across a waveguide excited in TE_{01} mode.

$$Y_r = \frac{\pi r^2 \epsilon_0}{b} \cdot \frac{1}{\frac{\nu}{\omega_p^2} + j\frac{\omega}{\omega_p^2} + \frac{1}{2j\omega}}.$$

Hence the equivalent circuit is as shown in Fig. 2 where

$$R_g = \frac{\nu}{\omega_p^2} \cdot \frac{b}{\pi r^2 \epsilon_0}, \qquad L_g = \frac{1}{\omega_p^2} \cdot \frac{b}{\pi r^2 \epsilon_0},$$
$$C_g = \frac{2\pi r^2 \epsilon_0}{b}.$$

This resonates at $\omega = \omega_p/\sqrt{2}$ and the Q of the series tuned circuit alone is ω/ν. The circuit produces maximum reflection and minimum transmission at resonance. Defining the bandwidth of reflected power as frequency interval within which reflected power is greater than $\frac{1}{2}$ maximum, we obtain the ratio of center frequency to bandwidth

$$Q_R = \frac{1}{\frac{\nu}{\omega} + \frac{\pi r^2 k_0^2}{bk_g}} = \frac{1}{\frac{\nu}{\omega} + \frac{2\pi^2 r^2 \lambda_g}{b\lambda_0^2}}.$$

The second term in the denominator, the damping term, will usually be larger than ν/ω, typically in a standard S-band guide at 3 Gc/s it will be of the order of 0.1, giving $Q_R \approx 10$. The effect of glass walls is small. (See Bryant and Franklin.[3])

While it may be possible to obtain larger Q_R by suitable dimensional adjustments, it should be observed that at resonance, the reflected power will be proportional to $[1 - Q_R(\nu/\omega)]^2$ and the transmitted power to $[Q_R(\nu/\omega)]^2$. It may be difficult to make ω/ν greater than 1000; hence, if Q_R were made, *e.g.*, 100, only 80 per cent or so of power would be reflected at resonance, and nearly 20 per cent lost in heating the plasma. This would get worse if Q_R were increased still further.

Looking at the transmitted signal, we can define the bandwidth either as the frequency interval within which the attenuation is more than half maximum leading to Q_A, or the interval within which the transmitted power is less than twice minimum leading to Q_T.

It can be shown that $Q_A = Q_R$ while

$$Q_T = \frac{\omega}{\nu}\left\{1 - 2\left(\frac{bk_g}{\pi r^2 k_0^2} \cdot \frac{\nu\omega}{\omega_p^2}\right)^2\right\},$$

which for very small collision frequency ($\nu \ll \omega$) tends to ω/ν.

Finally we would like to point out that the term Q is used somewhat loosely both here and by other authors. The loaded Q of the circuit in Fig. 2 calculated on the basis of stored and dissipated energy comes to

$$\frac{1}{\frac{\nu}{\omega}\left(1 + 2\frac{bk_g}{\pi r^2 k_0^2} \cdot \frac{\nu\omega}{\omega_p^2}\right)}$$

which is like neither Q_R, Q_A nor Q_T.

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Intrinsic Attenuation

In connection with a recent paper by Beatty,[1] we wish to advise that we have found a general method of decomposition of the two-arm dissipative waveguide junctions taking into account the intrinsic attenuation.[2]

We have defined for any two-arm junction D_1 a family of associated two-arm junctions so that each of its element D_2 verifies

$$q_1 d_1 q_2 = d_2$$

q_1, q_2, d_1, d_2 being the wave transformation matrices of the two-arm junctions Q_1Q_2 D_1D_2 (Q_1, Q_2 being nondissipative). Moreover, to study a family, we have found it easier to consider the matrices

$$d_r = \begin{vmatrix} c_{11}^* & -c_{21}^* \\ -c_{12}^* & c_{22}^* \end{vmatrix}$$

[1] R. W. Beatty, "Intrinsic attenuation," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-11, pp. 179–182; May, 1963.

[2] "Classification des quadripoles dissipatifs en hyperfréquence," *Compt. rend. acad. sci.*, vol. 257, p. 3576; 1963.